

3.12: Determine whether G defined by

$$G := \left\{ \begin{pmatrix} x & y \\ 0 & 1 \end{pmatrix} \mid x, y \in R, x \neq 0 \right\}$$

is or is not unimodular.

Solution: We will show that G is not unimodular, i.e., has a left-Haar measure that is not equal to its right-Haar measure. First, we record that

$$\begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x & y \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} xa & ay + b \\ 0 & 1 \end{pmatrix} \tag{1}$$

while

$$\begin{pmatrix} x & y \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} xa & bx + y \\ 0 & 1 \end{pmatrix}. \tag{2}$$

Do a bunch more, then, you are done. ★

3.13: This is just another problem.

Solution: I don't know how to do this one. ★