Math 146 Midterm Project, Spring 2023
Applications of the Integral

When is this due? **Friday, April 21, 5 pm** (I can be a little flexible if necessary, but need to see your work no later than Sunday morning in order to submit midterm grades on time.)

Purpose of this assignment: To learn some more about applications of the integral in "real life" and to practice reporting out results using computations, graphs and paragraphs.

What to do: The calculus in each of these applications is not hard, but understanding the situations might require some time and thought. Please feel free to ask me questions; **you shouldn't be talking to anyone else about this work**. Your only other resources should be:

- Desmos, Mathematica, Google Sheets, and/or Excel
- the resources linked on Moodle
- your class notes
- the textbook
- the resources linked in Part 2.

If you aren't sure whether or not a resource is allowed, please ask!

Please write up your results on paper as legibly as you can; scan to pdf and upload on Moodle. Include explanations in complete sentences. See Grading Criteria below for expectations.

- **Part 1: pollution measurement and control**

  One of the earliest pollution problems brought to the attention of the US Environmental Protection Agency (EPA) was the case of Sioux Lake in eastern South Dakota. For years, a small paper plant nearby had been discharging waste containing carbon tetrachloride (CCl_4) into the lake. At the time the EPA learned of the problem, the chemical was entering the lake at a rate of 16 cubic yards/year.

  The agency immediately ordered the installation of filters designed to slow, and eventually stop, the flow of CCl_4 from the mill. Implementation of this project took 3 years, during which the flow of pollutant was constant at 16 cubic yards/year. Once the filters were installed, the flow declined until it stopped entirely. During this time span, the rate of flow was approximately $R(t) = t^2 - 14t + 49$ cubic yards/year, where t is the number of years since the EPA discovered the problem. (Hence, $t \geq 3$.)

  Write a brief report as if you were working for the EPA; your report should include:

  (a) A graph (drawn by hand is fine, with help from Desmos or Mathematica as needed) of the rate of CCl_4 flow, beginning at the time the EPA first learned of the situation.

  (b) A discussion of how many years elapsed between the time the EPA learned of the situation and the time the pollution flow stopped entirely.

(c) A discussion of how much CCl_4 entered the waters during the time shown in your graph in part (a). (Don't forget units.)

- **Part 2: Lorenz curves and the Gini Coefficient**
  The Gini coefficient measures economic inequality using integration and a curve known as the Lorenz curve. You'll be choosing a country, approximating its Lorenz curve, and using it to find the Gini coefficient. For this report, you may assume that your audience knows as much as you do about these topics, but no more.
    - Watch this video for some background. You'll only need the first 7 minutes to get the idea.
    - Next, go to this website to find Lorenz curves for various countries around the world. Choose a country that interests you, and send me an email to claim it before you do any further work. (I will keep this document updated with a list of countries that have already been claimed. Because I use the US in my demo video below, that country is not available. Croatia, Cyprus, Finland, and the Netherlands have also been claimed.)
    - See this video for guidance on using Google Sheets to turn your Lorenz curve data into a 4th-degree polynomial. (I made the video for a previous course, but the information is still good!)
    - Use your 4th-degree polynomial to estimate the Gini coefficient for your country.
    - Write a brief report that includes the formula for your approximate Lorenz curve, the integral you used to find the Gini coefficient, and your final result. Comment on whether "your" country has a high or low degree of inequality.

- **Part 3: Probability density functions**
  Imagine that you are working as an efficiency consultant for a grocery store; they'd like most of their customers to take no longer than 7 minutes for the entire process of checking out, and want your advice to know whether they need to hire more cashiers. Your team observes a few days' worth of data, and is confident that the probability density function below is a good model for this situation, but wants you to finish the work and make a recommendation to the store management. To do this, carry out the tasks below..

  Let x denote the time (in minutes) that a person spends waiting in a checkout line and y the time (in minutes) that it takes to check out. Suppose the joint probability density for x and y is

$$f(x, y) = \frac{1}{8}e^{-x/4-y/2}.$$

  Do the following with this density function:

- Find the probability that a person spends between 0 and 4 minutes waiting in line, and then 0 to 3 minutes checking out. Show the steps carefully in evaluating your integral.
- Explain why the probability you found above doesn't really completely answer the question posed by the store. (Hint: could you wait in line for 5 minutes and still get out of the store in fewer than 7 minutes?) Given this, if the store wants at least half of customers to spend no more than 7 minutes total waiting in line and checking out, do they need to change anything? Explain.
- Suppose you want to set up (but not evaluate) an iterated integral that will answer the question precisely–that is, the integral whose value determines the probability that a person spends at most 7 minutes total both waiting in line and checking out. Draw the region of integration for this integral. (You don't need to actually set up the integral or evaluate it unless you're enjoying this challenge–just show the region you'd integrate over.)

Grading criteria (note that any project that meets the "good faith effort" standard or higher may be resubmitted with revisions for a higher grade):

Criteria for a "good faith effort" (see the syllabus for criteria for a C in the course)**:**

- Nearly all of the project has been completed, but a small part has been skipped or overlooked, OR
- Work is complete, but there is one major error, multiple minor errors, or a lack of explanation that suggest that the author still needs more instruction on the topic.

Criteria for a B:

- Explanations and solutions are mostly correct, with few significant gaps or errors, AND
- Correct mathematical notation is used throughout, AND
- Any classmate who is familiar with these topics can understand what the author did and why they did it.

Criteria for an A: (think: "textbook example")

- Explanations and solutions are well-organized and totally correct, AND
- A classmate who is **un**familiar with these topics can understand what the author did and why they did it, AND
- Correct mathematical notation **and** vocabulary are used throughout.