Math 150 **SAMPLE FINAL** **Fall 2065**

Your Name

Your Signature

Student ID

- Give your answers in exact form. Do not give decimal approximations.
- Calculators are not allowed.
- In order to receive credit, you must show your work. Do not do computations in your head. Instead, write them out on the exam paper.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.

Problem	Total Points	Score
1	5	
2	5	
3	12	
4	8	
5	6	
6	9	
7	12	
8	18	
9	12	
10	8	
11	5	
Total	100	

1. [**5 points total**] Mark each statement below as true or false by circling **T** or **F**.

1. **T F** The function $f(x) = |x|$ is continuous at $x = 0$.

2. **T F** The graph of $f(x) = x^{2016}$ has an inflection point at $x = 0$.

3. **T F** If $x = a$ is a critical point of a function $f(x)$, then $f'(a) = 0$

4. **T F** If $f(x)$ and $g(x)$ are continuous functions which are defined for all real numbers, then

$$\int_a^b (cf(x) - g(x))dx = c\int_a^b f(x)dx + \int_b^a g(x)dx$$

5. **T F** If $f(y)$ is a twice differentiable function whose first derivative is continuous, decreasing, and negative for all real numbers y, the $f(y)$ is concave up.

2. [**5 points total**] Circle the correct answer.

1. Suppose f is a function such that $f'(2) = 5$ and $f(2) = 7$. An equation of the line tangent to the curve $y = f(x)$ at the point $x = 2$ is:

A. $y = 2x + 7$ B. $y - 7 = 2(x - 5)$ C. $y = 5x - 3$ D. None of the above

2. Suppose f has a local maximum at a. What can you say about $f''(a)$?

A. $f''(a) > 0$. B. $f''(a) < 0$. C. $f''(a) = 0$. D. You cannot say anything about $f''(a)$ without more information.

3. Suppose that for $a < b < c$, $\int_a^b f(x)\,dx = 5$, $\int_b^c g(x)\,dx = 3$ and $\int_a^c g(x)\,dx = 7$. What is the value of $\int_a^b (f(x) - g(x))\,dx$?

A. -5 B. 0 C. 1 D. 9

4. Consider the function $h(x) = e^{-g(x)}$ where the function $g(x)$ is continuous with a continuous first derivative in $(-\infty, \infty)$. If the function $g(x)$ has a local maximum at the point a, then the function $h(x)$

A. Has local minimum at the point a.

B. Has local maximum at the point a.

C. Is negative at the point a.

D. You cannot say anything about $h(x)$ without more information.

5. Suppose f is a function such that $f'(3) = 0$, and $f''(3) = 0$. What can be said about the function?

A. The function has local maximum value at $x = 3$.

B. The function has local minimum value at $x = 3$.

C. The function has neither a local maximum nor local minimum value at $x = 3$.

D. You need more information to determine whether f has a local maximum or minimum at $x = 3$.

3. [**12 points total**] Consider the function $f(x) = \frac{e^x}{x-1}$.

(a) (3 pts) Find the x-coordinates of the x-intercepts and the y-coordinates of the y-intercept (if any).

(b) (3 pts) Find the intervals on which f increases and the intervals on which f decreases.

(c) (3 pts) Find the x-coordinates of any local maxima or minima.

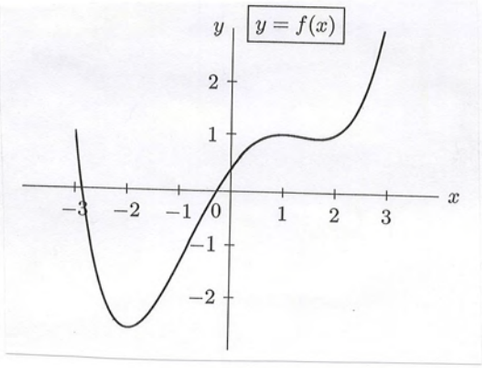
(d) (3 pts) Find the intervals on which f is concave up and the intervals on which f is concave down.

4. [**8 points total**] For what value of c is

$$f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2cx, & x \geq 3 \end{cases}$$

continuous at every x?

5. [**6 points total**] Below is the graph of a function $f(x)$.



Graph its derivative $f'(x)$.

6. [**9 points total**] Evaluate the following limits. Show work!

(a) $\lim_{x\to 1} \dfrac{x^2-1}{|x-1|}$

(b) $\lim_{x\to 0^+} x \ln x$

(c) $\lim_{x\to 0} \dfrac{\int_0^x e^{-t^2}\,dt}{x}$

7. [**12 points total**] Find $f'(x)$ (you should simplify and write your final answers without negative exponents) if

(a) $f(x) = x^3 \ln 3x$

(b) $f(x) = \dfrac{\cos x}{x^4 + 3}$

(c) $f(x) = \sin e^{\tan x^2}$

(d) $f(x) = x^{\tan x}$

8. [**18 points total**] Evaluate the following integrals

(a) $\displaystyle\int_0^1 (x^2+2)\sqrt{x^3+6x+5}\,dx$

(b) $\displaystyle\int \frac{\sin x}{\cos^2 x}\,dx$

(c) $\displaystyle\int \frac{6x^5-\sqrt{x}+5x^2}{x^3}\,dx$

(d) $\displaystyle\int \frac{1}{1+16x^2}\,dx$

(e) $\displaystyle\int_0^{3\pi/2} |\sin x|\,dx$

(f) $\displaystyle\int \frac{(\ln x)^2}{x}\,dx$

9. [**12 points total**] Air is being pumped into a spherical balloon at the rate of 7 cubic centimeters per second. What is the rate of change of the radius at the instant the volume equals 36π? The volume of a sphere of radius r is $\frac{4\pi}{3}r^3$

10. [**8 points total**] Find the equation of the tangent line to the curve $e^y \sin x + x - xy = \pi$ at the point $(\pi, 0)$.

11. [**5 points total**] Show that the equation $3x + 2\cos x + 5 = 0$ has exactly one real root.