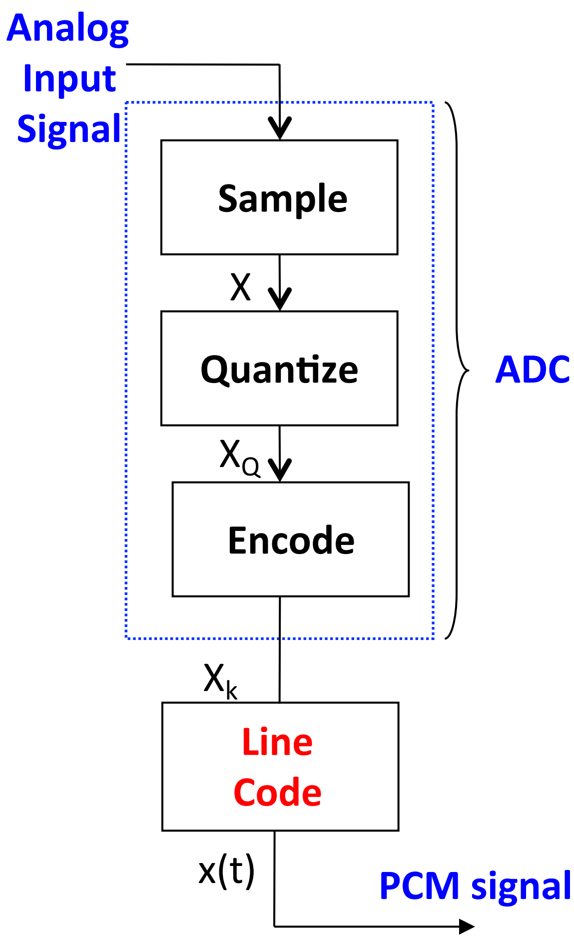
3.5. Line Codes and Spectra

Line Code in PCM



The output of an ADC can be transmitted over a baseband channel.

- The digital information must first be converted into a physical signal.
- The physical signal is called a ***line code***. Line coders use the terminology ***mark*** to mean **binary one** and ***space*** to mean **binary zero**.

3.5. Line Codes and Spectra

Binary Line Coding

DEFINIATION: Binary 1's and 0's, such as in PCM signaling, may be represented in various serial-bit signaling formats called ***line codes***.

There are two major categories: *return-to-zero* (**RZ**) and *nonreturn-to-zero* (**NRZ**).

With RZ coding, the waveform returns to a zero-volt level for a portion (usually on-half) of the bit interval.

3.5. Line Codes and Spectra

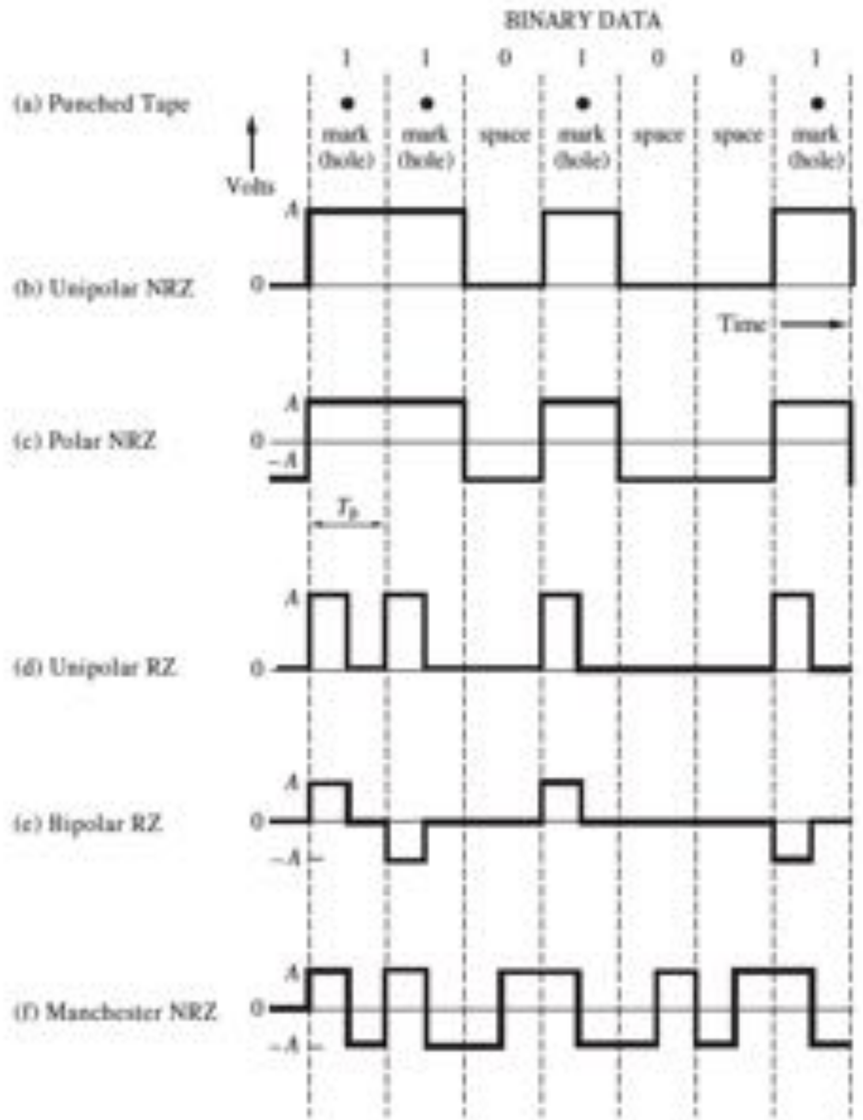


Figure 3–15 Binary signaling formats.

Unipolar signaling: "1" -- +A (high level)
"0" -- 0 (zero level)
Also called: on-off keying

polar signaling: "1" -- +A
"0" -- -A

Bipolar signaling: "1" -- alternately positive or negative values
"0" -- 0

Manchester signaling: "1" -- positive half-bit followed by a negative half-bit period pulse.
"0" – negative half-bit followed by a positive half-bit period pulse

3.5. Line Codes and Spectra

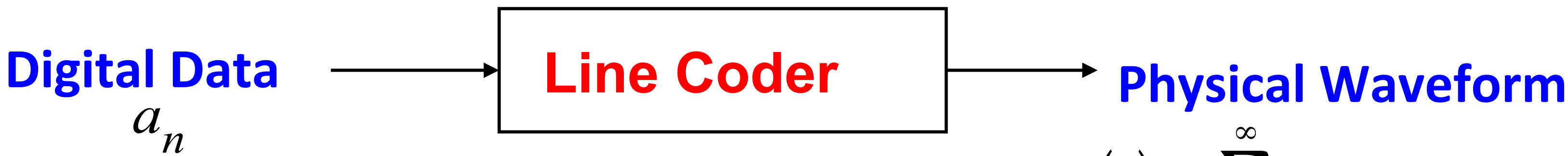
Binary Line Coding

Desirable properties of a line code:

- ***Self-synchronization:*** there is enough time information built in.
- ***Low probability of error:*** receivers can recover the binary data when the input data signals is corrupted.
- ***A spectrum that is suitable for the channel:*** the signal bandwidth needs to be sufficiently small compared to the channel bandwidth
- ***Transmission bandwidth:*** this should be as small as possible
- ***Error detection capability:*** this feature is easily implemented by the addition of channel encoders and decoders.
- ***Transparency:*** every possible sequence of data is faithfully and transparently received.

3.5. Line Codes and Spectra

Binary Line Coding



$$s(t) = \sum_{n=-\infty}^{\infty} a_n f(t - nT_s)$$

- The input to the line coder is a sequence of values, a_n that is a function of a data bit or an ADC output bit.

- The output of the line coder is a waveform $s(t) = \sum_{n=-\infty}^{\infty} a_n f(t - nT_s)$

Where $f(t)$ is the symbol pulse shape and T_s is the duration of one symbol. For binary signaling, $T_s = T_b$, where T_b is the time that it takes to send 1 bit. For multilevel signaling, $T_s = lT_b$. $\{a_n\}$ is the amplitude, "A" or "0" for NRZ, for example.

3.5. Line Codes and Spectra

Types of Line Codes

- Each line code is described by a ***symbol mapping function*** a_n, and a ***pulse shape*** *p(t)* through

$$s(t) = \sum_{n=-\infty}^{\infty} a_n f(t - nT_s)$$

- Categories of line codes:
  - Symbol mapping functions (a_n):
    - Unipolar
    - Polar
    - Bipolar
  - Pulse shape (*p(t)*):
    - NRZ (Nonreturn-to-zero)
    - RZ (Return to Zero)
    - Manchester (split phase)

3.5. Line Codes and Spectra

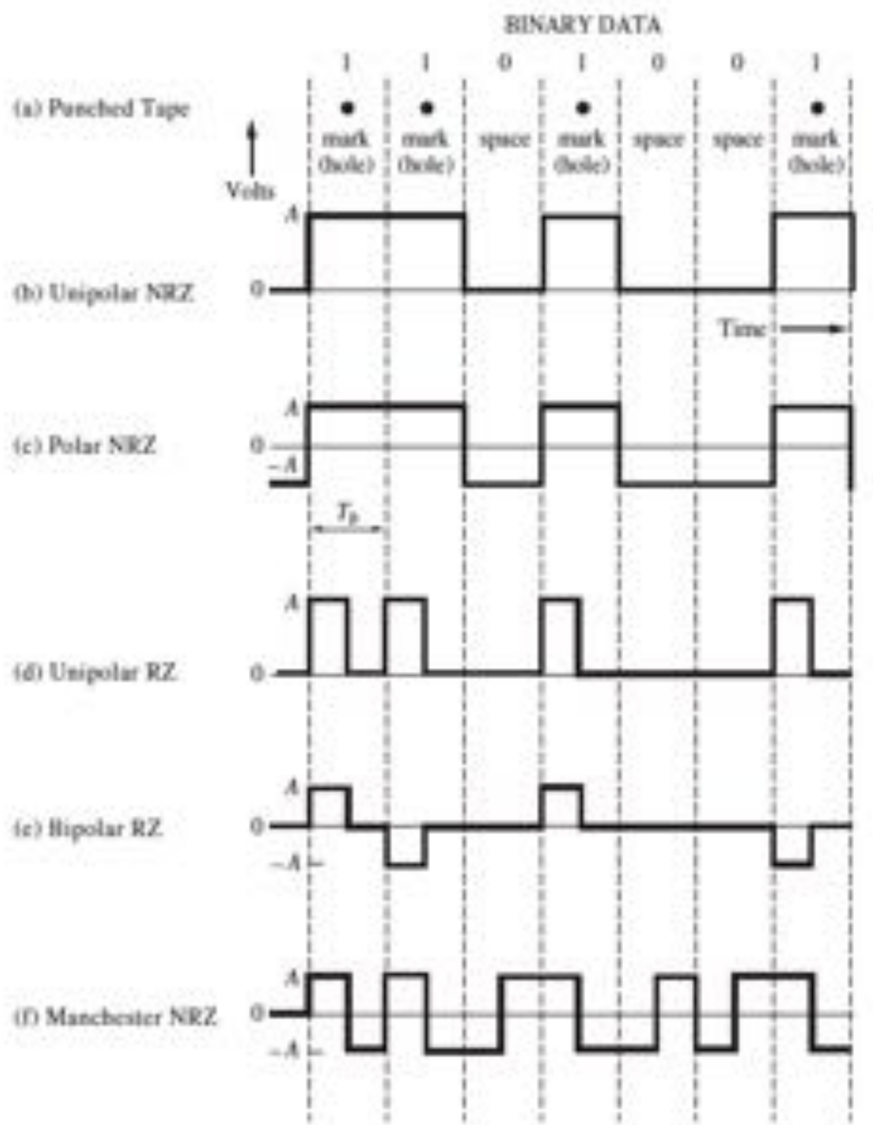


Figure 3–15 Binary signaling formats.

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"0" – negative half-bit followed by a positive half-bit period pulse

3.5. Line Codes and Spectra

Power Spectra for Binary Line Code

✧ A digital signal is represented by:

$$s(t) = \sum_{n=-\infty}^{\infty} a_n f(t - nT_s)$$

where a_n is the digital number (e.g. "0" or "1"), and f(t) is the symbol pulse shape (e.g. $f(t) = \Pi\left(\frac{t}{T_s}\right)$ For unipolar NRZ)

✧ **PSD** can be calculated using the autocorrelation function, and it depends on:

1.) the pulse shape
2.) statistical properties of data expressed by the autocorrelation function

3.5. Line Codes and Spectra

Power Spectra for Binary Line Code

The general expression for the **PSD of a digital signal** is:

$$p_s(f) = \frac{|F(f)|^2}{T_s} \sum_{k=-\infty}^{\infty} R(k) e^{j2\pi k f T_s}$$

Where $F(f)$ is the Fourier transform of the pules shape $f(t)$ and $R(k)$ is the autocorrelation of the data.

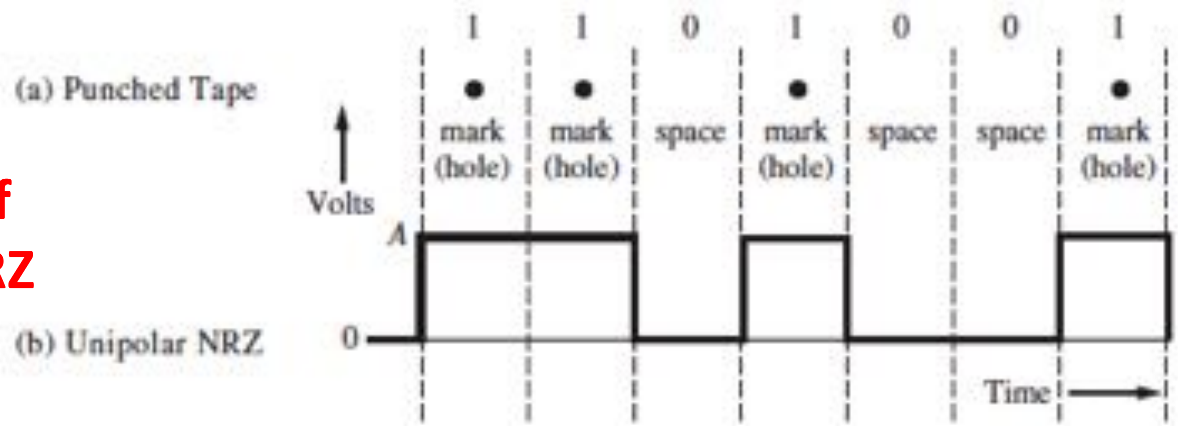
$$R(k) = \sum_{i=1}^{I} (a_n a_{n+k})_i P_i$$

Where a_n and a_{n+k} are the (voltage) levels of the data pulses at the n^{th} and $(n+k)^{th}$ symbol positions, respectively. P_i is the probability of having the i^{th} $a_n a_{n+k}$ product.

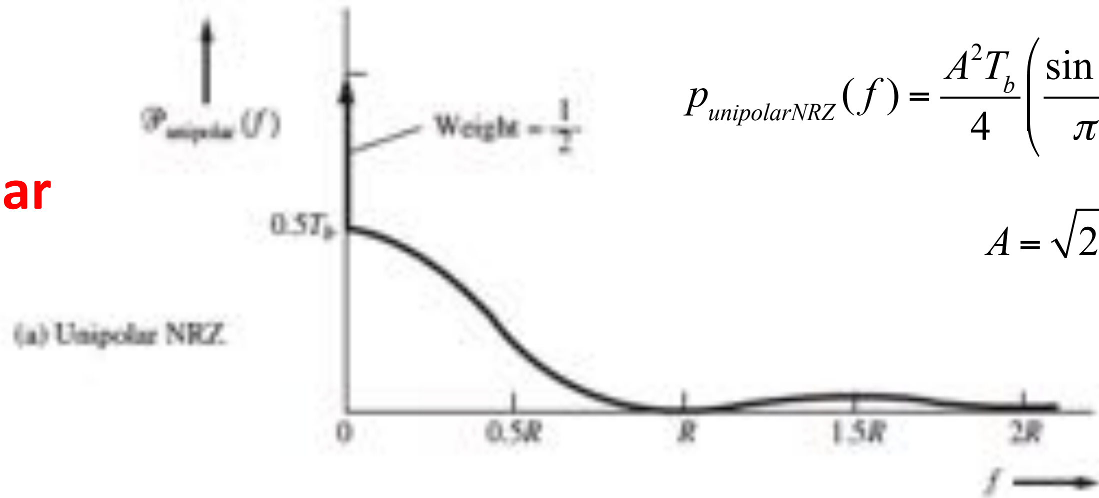
3.5. Line Codes and Spectra

PSD for Unipolar NRZ signaling

Line Code of Unipolar NRZ



PSD of Unipolar NRZ



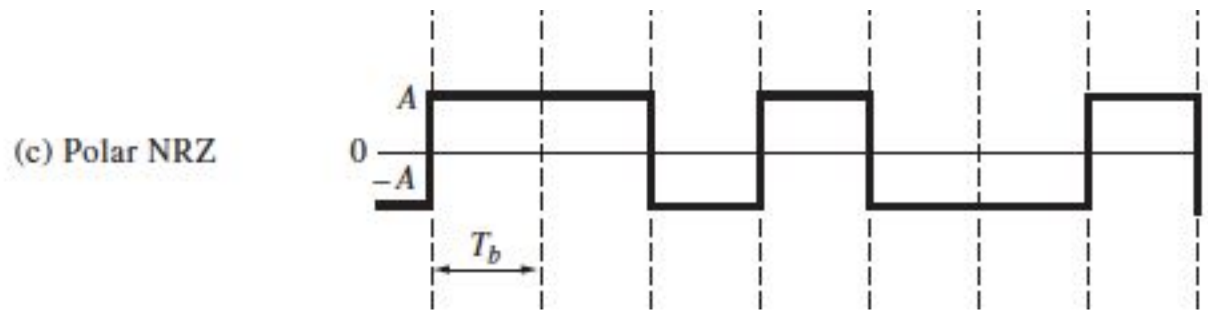
$$p_{unipolarNRZ}(f) = \frac{A^2 T_b}{4}\left(\frac{\sin \pi f T_b}{\pi f T_b}\right)^2 \left[1 + \frac{1}{T_b}\delta(f)\right]$$

$$A = \sqrt{2} \qquad 1/T_b = R$$

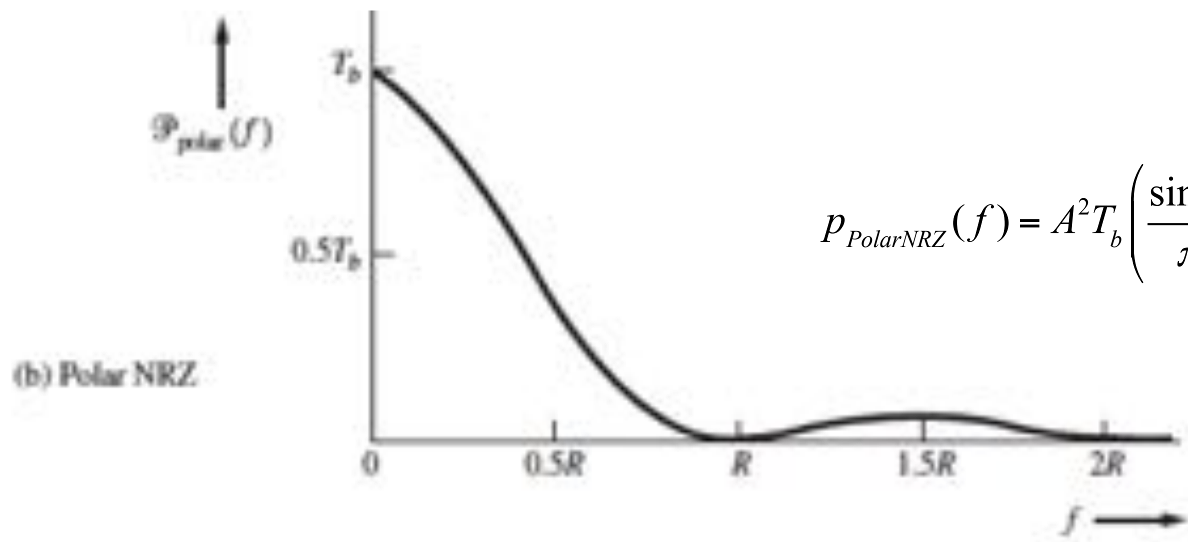
3.5. Line Codes and Spectra

PSD for Polar NRZ signaling

Line Code of Polar NRZ



PSD of Polar NRZ



$$p_{PolarNRZ}(f) = A^2 T_b \left(\frac{\sin \pi f T_b}{\pi f T_b} \right)^2$$

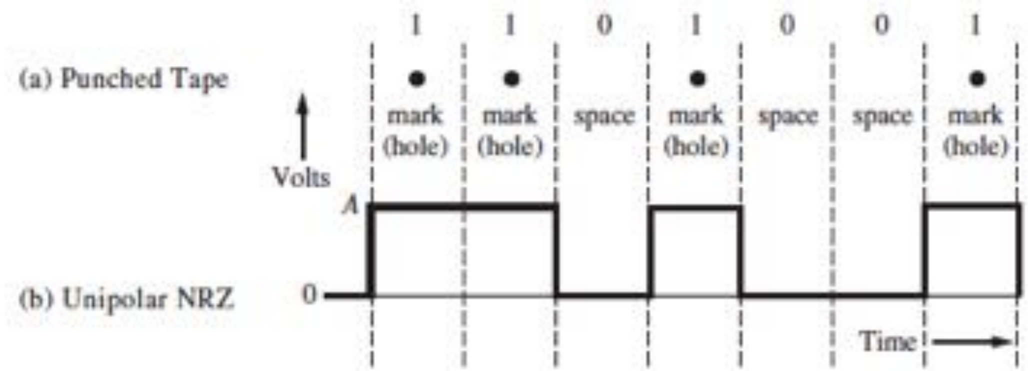
3.5. Line Codes and Spectra

Differential Coding

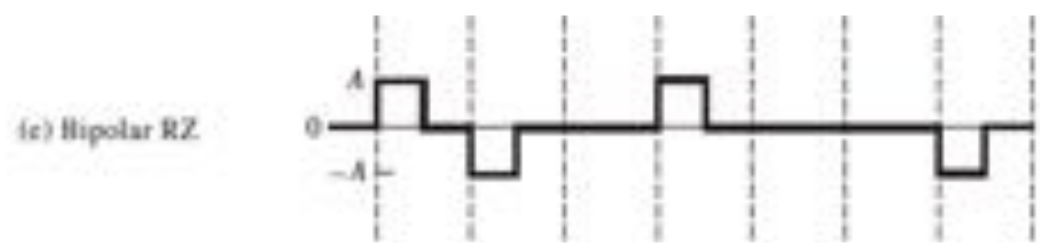
Potential issues is unipolar NRZ, polar NRZ, and Manchester NRZ:

✧ The waveform is often ***unintentionally inverted*** (happens in a twisted-pair transmission line channel by switching the two leads)

✧ **Results: 1 - > 0, 0 - > 1**



✧ This is not a issue for bipolar signal



3.5. Line Codes and Spectra

Differential Coding

The technology called differential coding can be used to ameliorate this problem.

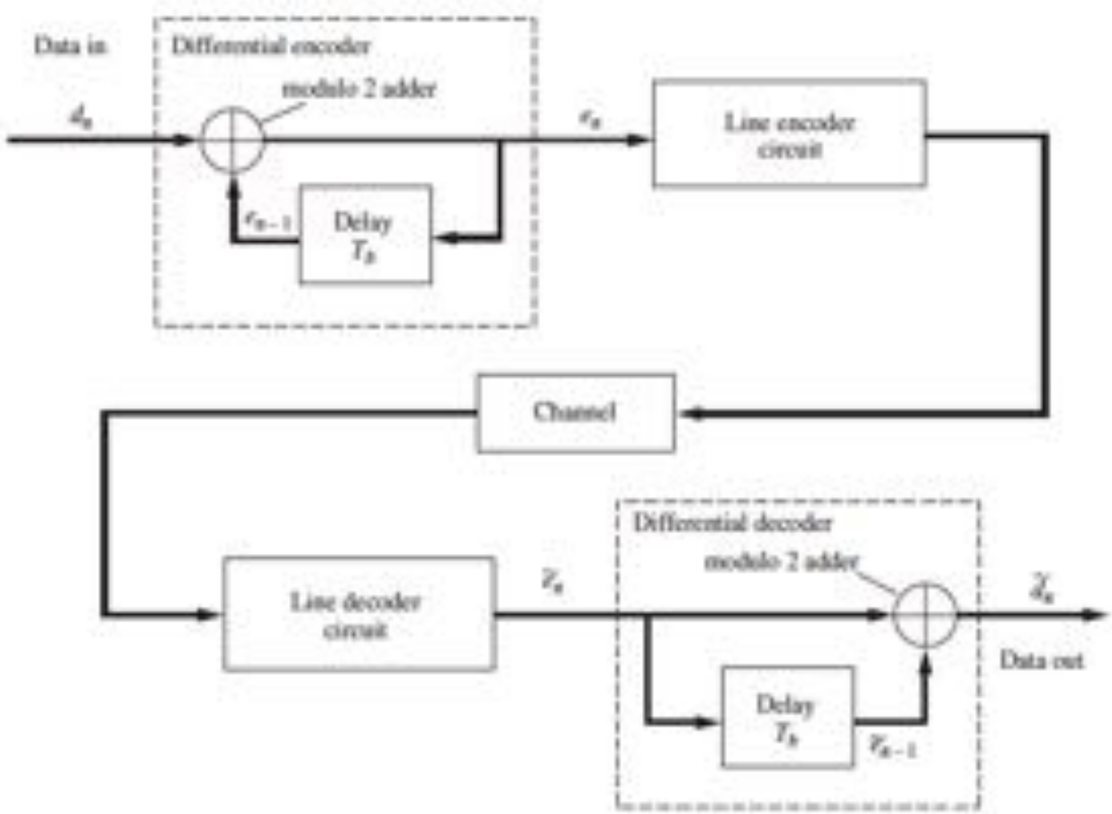


Figure 3–17 Differential coding system.

$$e_n = d_n \oplus e_{n-1}$$

$$\tilde{d}_n = \tilde{e}_n \oplus \tilde{e}_{n-1}$$

Where $\oplus$ is a modulo 2 adder or an exclusive-OR gate (XOR) operation.

3.5. Line Codes and Spectra

Differential Coding

Encoding $e_n = d_n \oplus e_{n-1}$

Input sequence	d_n		1	1	0	1	0	0	1
Encoded sequence	e_n	1	0	1	1	0	0	0	1
Reference digit		↑							

Decoding (with correct channel polarity) $\tilde{d}_n = \tilde{e}_n \oplus \tilde{e}_{n-1}$

Receiver sequence (Correct polarity)	1	0	1	1	0	0	0	1
Decoded sequence		1	1	0	1	0	0	1

Decoding (with inverted channel polarity)

Received sequence (Inverted polarity)	0	1	0	0	1	1	1	0
Decoded sequence		1	1	0	1	0	0	1

3.5. Line Codes and Spectra

Regenerative Repeaters

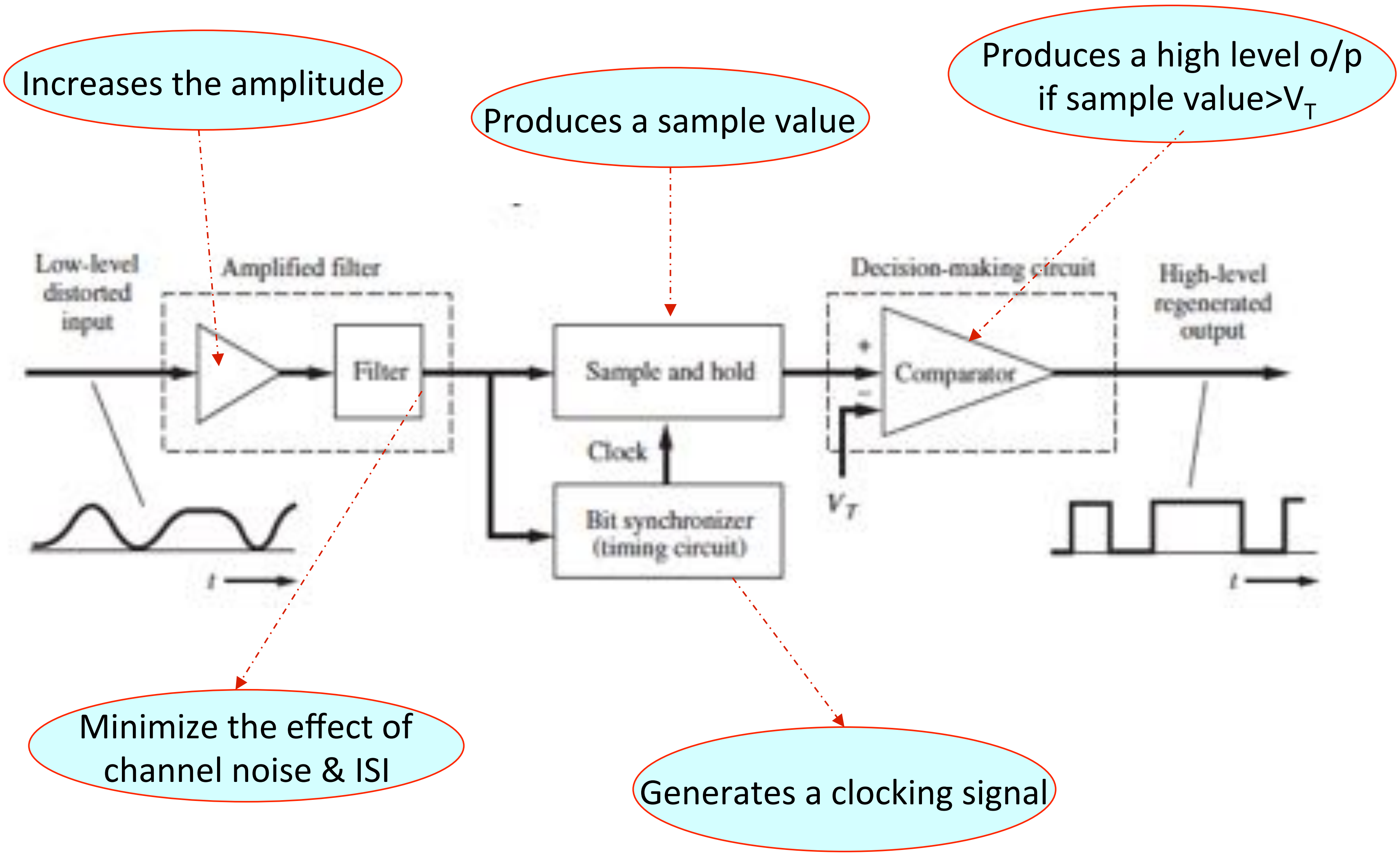
When a signal (waveform) is transmitted over a hardwire channel , it is filtered, attenuated, and corrupted by noise. Consequently, for long distance, the data cannot be recovered at the receiving end unless ***repeaters*** are utilized.

✧ For analog signal (such as PAM), only linear amplifiers with appropriate filter could be used. → only increase the **amplitude,** the in-band distortion (such as random noise) could accumulate.

✧ For digital signal (such as PCM), nonlinear processing can be used to regenerate a “noise-free” digital signal. This type of nonlinear processing is called a ***regenerative repeater***.

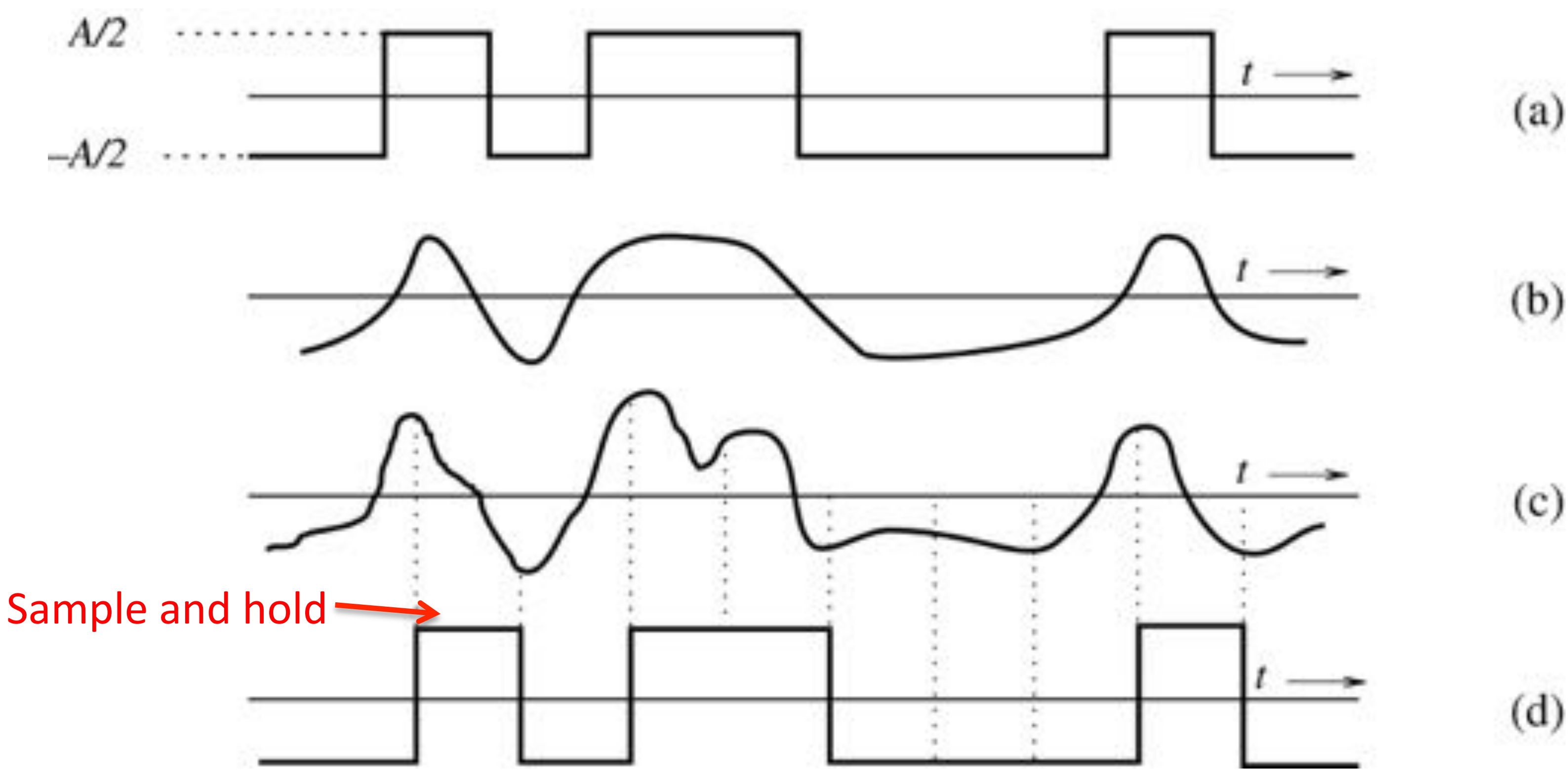
3.5. Line Codes and Spectra

Regenerative Repeaters



3.5. Line Codes and Spectra

Regenerative Repeaters



(a) Transmitted signal. (b) Received distorted signal (without noise).
(c) Received distorted signal (with noise). (d) Regenerated signal (delayed).

3.5. Line Codes and Spectra

Regenerative Repeaters

- In long-distance digital communication systems, many repeaters may be used in cascade, and the distance between the repeaters is governed by the path loss of the transmission medium and the amount of noise that is added.

- A repeater is required when the SNR at a point along the channel becomes lower than the value that is needed to maintain the overall probability-of-bit-error specification.

- For m repeaters in cascade, the overall probability of ***bit errors*** P_{me} is

$$P_{me} = mP_e$$

where P_e is the probability of bit error for a single repeater

3.5. Line Codes and Spectra

Bit Synchronization

✧ Synchronization signal are clock-type signals that are necessary within a receiver (or repeater) for detection (or regeneration) of the data from the corrupted input signal.

✧ These clock signals have precise frequency and phase relationship with respect to the received input signal, and they are delayed compared to the clock signals at the transmitter.

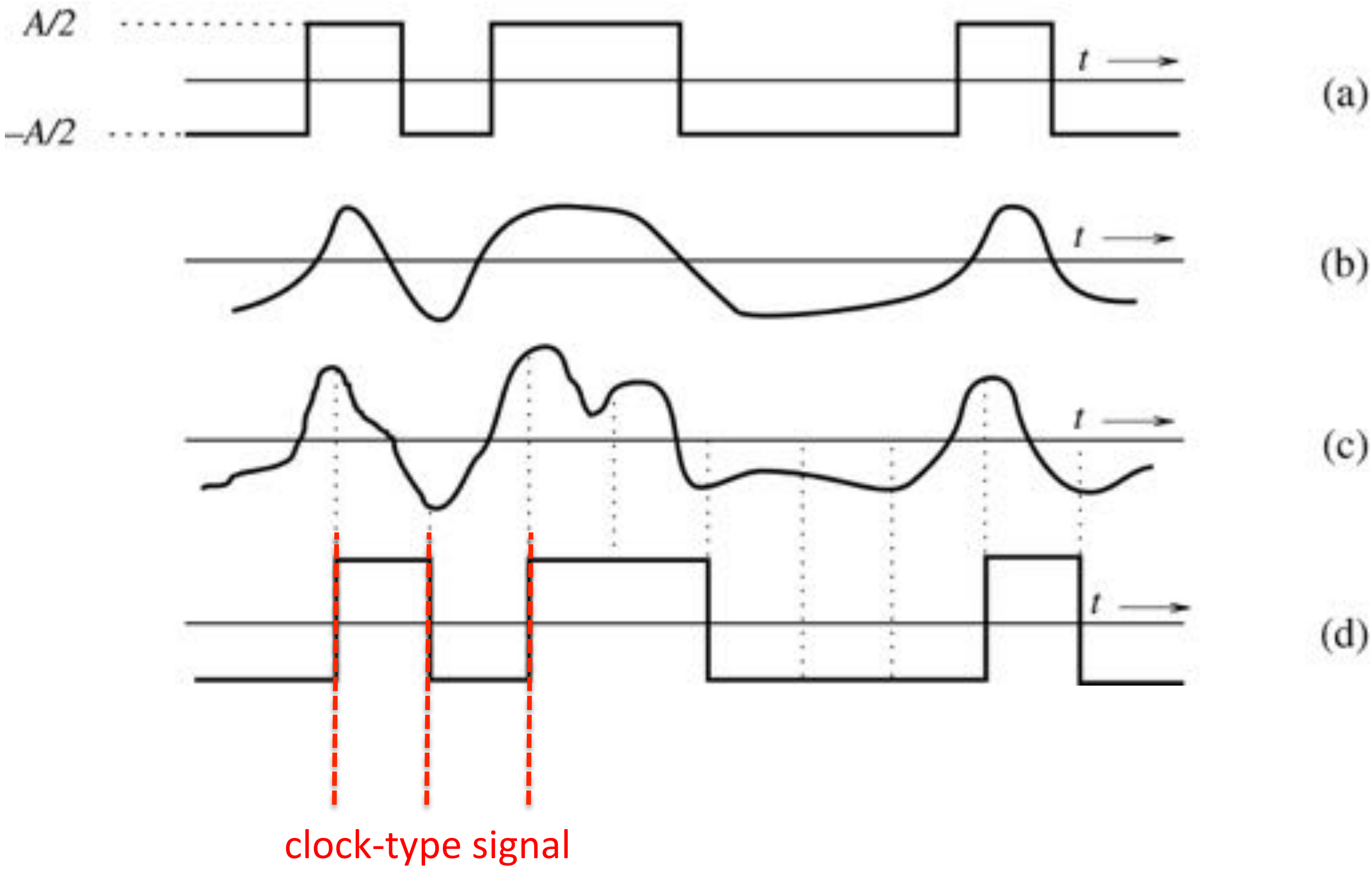
✧ At lease three types of synchronization signals are needed:

- *bit sync*
- *frame sync*
- *carrier sync*

3.5. Line Codes and Spectra

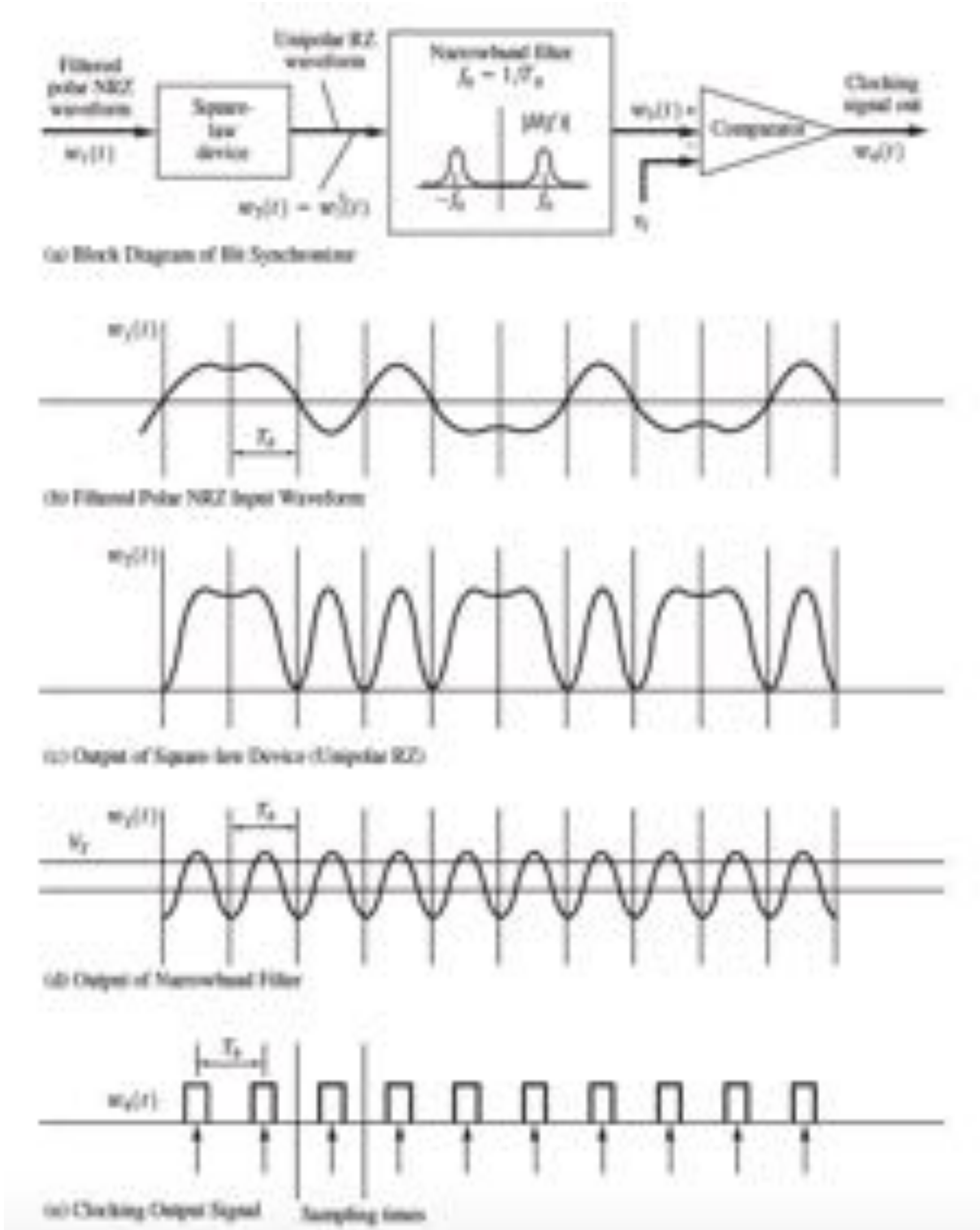
Bit Synchronization

Why sync signal is necessary??



3.5. Line Codes and Spectra

Square-law bit synchronizer for NRZ signal



3.5. Line Codes and Spectra

Power Spectra for Multilevel Polar NRZ signals

Why do we need multilevel signal?

$$\mathrm{w(t)} = \sum_{k=1}^{N} w_k \varphi_k(t) \qquad 0<t<T_0$$

w_k is the digital number (e.g. "0" and "1"), T_0 is the time period to send out a message

For **binary signal**: ***N*** (dimension number) = *n* (bit number)

For **multilevel signal**: ***N*** (dimension number) = *n/l* (bit number)

3.5. Line Codes and Spectra

Power Spectra for Multilevel Polar NRZ signals

Binary to multilevel conversion is used to reduce the bandwidth required by the binary signaling.

- ✧ Multiple bits (l number of bits) are converted into words having SYMBOL durations $T_s = lT_b$ where the Symbol Rate or the BAUD rate $D = 1/T_s = 1/lT_b$
- ✧ The symbols are converted to a L level ($L = 2^l$) multilevel signal using an l-bit DAC.
- ✧ Note that now the Baud rate is reduced (D=R/l), thus the bandwidth is also reduced.

3.5. Line Codes and Spectra

Power Spectra for Multilevel Polar NRZ signals

Bandwidth (minimum) of the waveform representing the digital signal:

$$B = \frac{N}{2T_0} = \frac{1}{2}D \qquad \text{Hz}$$

For binary signal: $$B = \frac{N}{2T_0} = \frac{1}{2}D$$

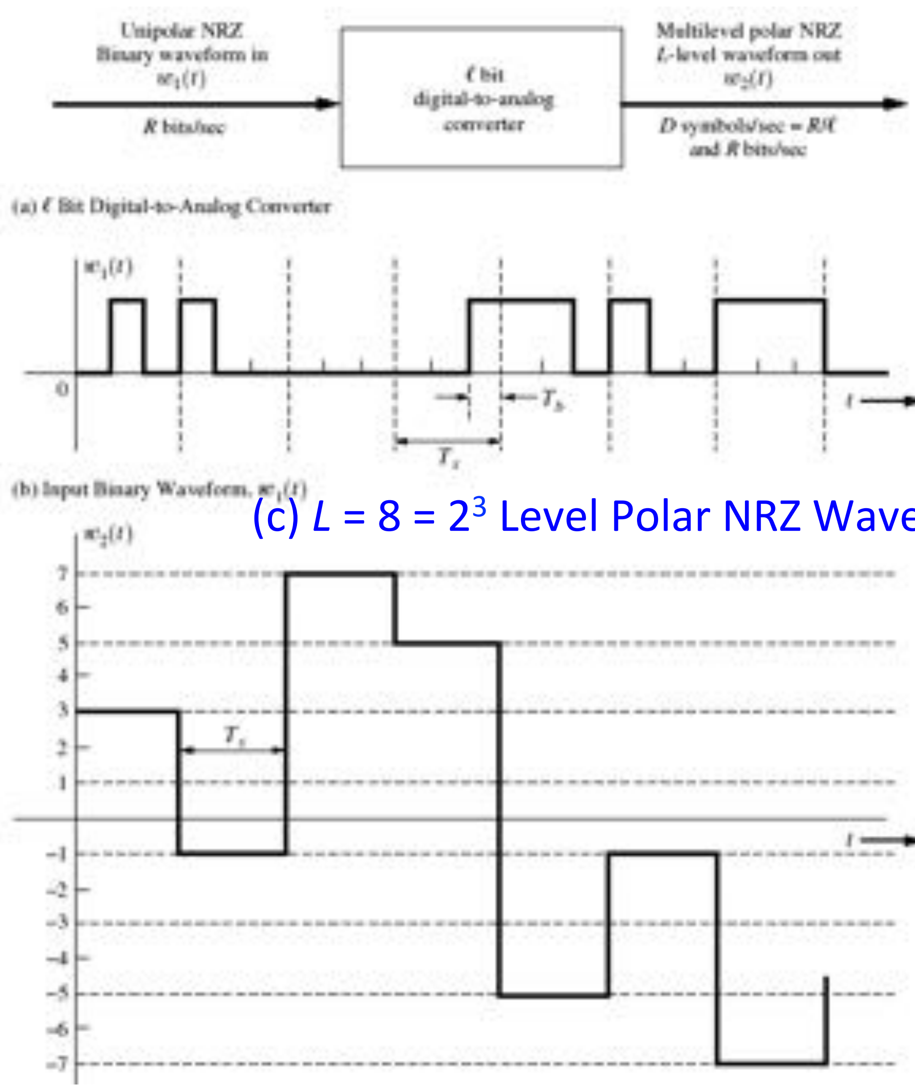
For multilevel signal: $$B = \frac{N}{2T_0 L} = \frac{1}{2L}D$$

3.5. Line Codes and Spectra

Power Spectra for Multilevel Polar NRZ signals

TABLE 3-5 THREE-BIT DAC CODE

Digital Word	Output Level, $(a_n)_k$
000	+7
001	+5
010	+3
011	+1
100	−1
101	−3
110	−5
111	−7



(c) $L = 8 = 2^3$ Level Polar NRZ Waveform Out

3.5. Line Codes and Spectra

Spectral Efficiency

DEFINATION: The spectral efficiency of a digital signal is given by the number of bits per second of data that can be supported by each hertz of bandwidth.

$$\eta = \frac{R}{B} \qquad \frac{(\text{Bit/s})}{\text{Hz}}$$

$$\eta_{\max} = \frac{C}{B} = \log_2\left(1 + \frac{S}{N}\right)$$

TABLE 3-6 SPECTRAL EFFICIENCIES OF LINE CODES

Code Type	First Null Bandwidth (Hz)	Spectral Efficiency $\eta = R/B$ [(bits/s)/Hz]
Unipolar NRZ	R	1
Polar NRZ	R	1
Unipolar RZ	$2R$	$\frac{1}{2}$
Bipolar RZ	R	1
Manchester NRZ	$2R$	$\frac{1}{2}$
Multilevel polar NRZ	R/ℓ	ℓ